

Evaluate  $\int (5x^2 - x + 3) \sin 2x \, dx$ . =  $\left(-\frac{5}{2}x^2 + \frac{1}{2}x - \frac{3}{2}\right) \cos 2x$   $\textcircled{2}$  SCORE: \_\_\_\_ / 5 PTS

$$\begin{array}{r}
 \underline{u} \\
 5x^2 - x + 3 \\
 10x - 1 \\
 10 \\
 0
 \end{array}
 \begin{array}{l}
 \underline{dv} \\
 \sin 2x \\
 + \\
 -\frac{1}{2} \cos 2x \\
 - \\
 -\frac{1}{4} \sin 2x \\
 + \\
 \frac{1}{8} \cos 2x
 \end{array}$$

$$\textcircled{\frac{1}{2}} + \left(\frac{5}{2}x - \frac{1}{4}\right) \sin 2x + \frac{5}{4} \cos 2x \textcircled{1} + C$$

$$= \left(-\frac{5}{2}x^2 + \frac{1}{2}x - \frac{1}{4}\right) \cos 2x + \left(\frac{5}{2}x - \frac{1}{4}\right) \sin 2x + C \textcircled{\frac{1}{2}}$$

$\textcircled{-\frac{1}{2}}$  IF YOU FORGOT  $+C$

Evaluate  $\int e^{-2x} \cos 5x \, dx$ . =  $\underline{-\frac{1}{2} e^{-2x} \cos 5x + \frac{5}{4} e^{-2x} \sin 5x}$  ②

SCORE: \_\_\_\_ / 5 PTS

$$\begin{array}{rcl} \underline{u} & \underline{dv} & \\ \cos 5x & + & e^{-2x} \\ -5 \sin 5x & \swarrow & -\frac{1}{2} e^{-2x} \\ -25 \cos 5x & \searrow & + \frac{1}{4} e^{-2x} \end{array}$$

$\underline{-\frac{25}{4} \int e^{-2x} \cos 5x \, dx}$  ①

$\underline{\frac{29}{4} \int e^{-2x} \cos 5x \, dx = -\frac{1}{2} e^{-2x} \cos 5x + \frac{5}{4} e^{-2x} \sin 5x}$  ①

$\underline{\int e^{-2x} \cos 5x \, dx = -\frac{2}{29} e^{-2x} \cos 5x + \frac{5}{29} e^{-2x} \sin 5x + C}$  ①

①  $-\frac{1}{2}$  IF YOU FORGOT  $+C$

Evaluate  $\int \sec^6 x \tan^4 x \, dx$ .  $= \int \sec^4 x \tan^4 x \sec^2 x \, dx$

SCORE: \_\_\_\_ / 5 PTS

$v = \tan x$  ①  
 $dv = \sec^2 x \, dx$

$= \int (v^2 + 1)^2 v^4 \, dv$  ①

$= \int (v^8 + 2v^6 + v^4) \, dv$  ①

$= \frac{1}{9} v^9 + \frac{2}{7} v^7 + \frac{1}{5} v^5 + C$  ①

$= \frac{1}{9} \tan^9 x + \frac{2}{7} \tan^7 x + \frac{1}{5} \tan^5 x + C$  ①

①  $-\frac{1}{2}$  IF YOU FORGOT  
+C AT THE  
END



Prove the reduction formula  $\int \cos^n u \, du = \frac{1}{n} \cos^{n-2} u \sin u + \frac{n-1}{n} \int \cos^{n-2} u \, du$ .

SCORE: \_\_\_\_ / 6 PTS

NOTE: You must show how to get this formula.

You will receive 0 credit if your "proof" is differentiating both sides of the equation.

$$\begin{array}{r} \frac{f}{\cos^{n-1} u} \quad \frac{g}{\cos u} \\ \quad \quad \quad + \\ -(n-1) \cos^{n-2} u \sin u \Rightarrow \sin u \end{array}$$

$$\int \cos^n u \, du = \cos^{n-1} u \sin u + (n-1) \int \cos^{n-2} u \sin^2 u \, du \quad (2)$$

$$= \cos^{n-1} u \sin u + (n-1) \int \cos^{n-2} u (1 - \cos^2 u) \, du \quad (1\frac{1}{2})$$

$$= \cos^{n-1} u \sin u + (n-1) \int \cos^{n-2} u \, du - (n-1) \int \cos^n u \, du \quad (1)$$

$$n \int \cos^n u \, du = \cos^{n-1} u \sin u + (n-1) \int \cos^{n-2} u \, du \quad (1)$$

$$\int \cos^n u \, du = \frac{1}{n} \cos^{n-1} u \sin u + \frac{n-1}{n} \int \cos^{n-2} u \, du \quad (1\frac{1}{2})$$

Evaluate  $\int (x^2 - 9)^{\frac{3}{2}} dx$ .

SCORE: \_\_\_\_ / 9 PTS

$$x = 3 \sec \theta \quad \longrightarrow \quad \sec \theta = \frac{x}{3}$$



$$dx = 3 \sec \theta \tan \theta d\theta$$

$$\int (9 \sec^2 \theta - 9)^{\frac{3}{2}} \cdot 3 \sec \theta \tan \theta d\theta$$

$$= \int (9 \tan^2 \theta)^{\frac{3}{2}} \cdot 3 \sec \theta \tan \theta d\theta$$

$$= 81 \int \sec \theta \tan^4 \theta d\theta$$

$$= 81 \int \sec \theta (\sec^2 \theta - 1)^2 d\theta$$

$$= 81 \left[ \int \sec^5 \theta d\theta - 2 \int \sec^3 \theta d\theta + \int \sec \theta d\theta \right]$$

$$= 81 \left[ \frac{1}{4} \sec^3 \theta \tan \theta + \frac{3}{4} \int \sec^3 \theta d\theta - 2 \int \sec^3 \theta d\theta + \int \sec \theta d\theta \right]$$

$$= 81 \left[ \frac{1}{4} \sec^3 \theta \tan \theta - \frac{5}{4} \left( \frac{1}{2} (\sec \theta \tan \theta + \ln |\sec \theta + \tan \theta|) \right) + \ln |\sec \theta + \tan \theta| \right] + C$$

$$= 81 \left[ \frac{1}{4} \sec^3 \theta \tan \theta - \frac{5}{8} \sec \theta \tan \theta + \frac{3}{8} \ln |\sec \theta + \tan \theta| \right] + C$$

$$= 81 \left[ \frac{1}{4} \left( \frac{x}{3} \right)^3 \frac{\sqrt{x^2 - 9}}{3} - \frac{5}{8} \frac{x}{3} \frac{\sqrt{x^2 - 9}}{3} + \frac{3}{8} \ln \left| \frac{x}{3} + \frac{\sqrt{x^2 - 9}}{3} \right| \right] + C$$

$$= \frac{1}{4} x^3 \sqrt{x^2 - 9} - \frac{45}{8} x \sqrt{x^2 - 9} + \frac{243}{8} \ln |x + \sqrt{x^2 - 9}| + C$$

IF YOU FORGOT +C AT THE END